

stochastic models

- ③. A military maintenance depot overhauls tanks. There is room for three tanks in the facility and one tank in an overflow area outside. At most four tanks can be at the depot at one time. Every morning a tank arrives for an overhaul. If the depot is full, however, it is turned away, so no arrivals occur under these circumstances. ~~When the depot is full, the entire overhaul schedule is delayed 1 day.~~ On any given day, the following probabilities govern the completion of overhauls.

Number of tanks completed	0	1	2	3
Probability	0.2	0.4	0.3	0.1

These values are independent of the number of tanks in the depot, but obviously no more tanks than are waiting at the start of the day can be completed.

- (a) Develop a Markov chain model for this situation. Begin by defining the state to be the number of tanks in the depot at the start of each day (after the scheduled arrival). Draw the network diagram and write the state-transition matrix.
- (b) Do the same when the state is defined as the number of tanks in the depot at the end of each day.

$$\begin{aligned}
 S^i &= \# \text{ tanks in depot} \\
 &\quad \text{after start of day (arrival)} \\
 &= 1, 2, 3, 4
 \end{aligned}$$

note S can not be 0 since if depot is empty, arrival takes us to 1 and we then count

$$\tilde{P} = \begin{pmatrix} .8 & .2 & 0 & 0 \\ .4 & .4 & .2 & 0 \\ .1 & .3 & .4 & .2 \\ .1 & .3 & .4 & .2 \end{pmatrix}$$

$$\begin{aligned}
 P_{11} &= P(1 \text{ arrives and } 1 \text{ is finished}) \\
 P_{12} &= P(1 \text{ arrives and } 2 \text{ are finished}) \\
 P_{21} &= P(2 \text{ arrives and } 2 \text{ are finished}) \\
 P_{22} &= P(2 \text{ arrives and } 1 \text{ is finished}) \\
 P_{23} &= P(1 \text{ arrives and } 0 \text{ are finished}) \\
 P_{41} &= P(2 \text{ arrives but turned away to } 3 \text{ finished})
 \end{aligned}$$